Cyclic and Long-Term Variation of Sunspot Magnetic Fields

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Abstract Measurements from the Mount Wilson Observatory (MWO) were used to study the long-term variations of sunspot field strengths from 1920 to 1958. Following a modified approach similar to that presented in Pevtsov *et al.* (*Astrophys. J. Lett.* **742**, L36, 2011), we selected the sunspot with the strongest measured field strength for each observing week and computed monthly averages of these weekly maximum field strengths. The data show the solar cycle variation of the peak field strengths with an amplitude of about $500 - 700$ gauss (G), but no statistically significant long-term trends. Next, we used the sunspot observations from the Royal Greenwich Observatory (RGO) to establish a relationship between the sunspot ar-

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eas and the sunspot field strengths for cycles $15-19$. This relationship was used to create a proxy of the peak magnetic field strength based on sunspot areas from the RGO and the USAF/NOAA network for the period from 1874 to early 2012. Over this interval, the magnetic field proxy shows a clear solar cycle variation with an amplitude of $500-700$ G and a weaker long-term trend. From 1874 to around 1920, the mean value of magnetic field proxy increases by about $300-350$ G, and, following a broad maximum in $1920-1960$, it decreases by about 300 G. Using the proxy for the magnetic field strength as the reference, we scaled the MWO field measurements to the measurements of the magnetic fields in Pevtsov *et al.* (2011) to construct a combined data set of maximum sunspot field strengths extending from 1920 to early 2012. This combined data set shows strong solar cycle variations and no significant long-term trend (the linear fit to the data yields a slope of -0.2 ± 0.8 G year^{-1}). On the other hand, the peak sunspot field strengths observed at the minimum of the solar cycle show a gradual decline over the last three minima (corresponding to cycles $21-23$) with a mean downward trend of ≈ 15 G year^{-1}.

1. Introduction

Recently, several studies have concentrated on the long-term variations of field strengths in sunspots. The question at the core of these investigations was whether the strength of sunspot magnetic fields has gradually declined over the last two cycles.

Penn and Livingston (2006, 2011) measured the field strength using the separation of two Zeeman components of the magnetically sensitive spectral line Fe I 1564.8 nm over the declining phase of solar cycle 23 and the rising phase of cycle 24. The measurements were taken on a daily basis (in quarterly observing intervals due to telescope scheduling). In early years, only the large sunspots were measured; more recent observations are aimed at including all sunspots and pores that are present on the disk. Monthly averages of these measurements show a gradual decrease in sunspot field strength from the beginning of the project (late 1998) to the present (mid-2012). It is possible that the non-uniformity of the data set (*i.e.*, fewer measurements at the beginning, newer observations include both sunspots and pores) may result in such a decline in average field strengths. However, the most recent study by Livingston, Penn, and Svalgaard (2012) indicates that there is no change in the shape of the distribution of measured field strengths over the observing period; only the mean of the distribution changes. This constancy in the shape of the field distribution appears to rule out the speculation that the decline in field strengths could be explained by the non-uniformity of the data set.

Watson, Fletcher, and Marshall (2011) employed the magnetic flux measurements from the Michelson Doppler Imager (MDI) onboard the *Solar and Heliospheric Observatory* (SOHO), and studied the magnetic flux changes in sunspots over cycle 23 ($1996-2010$). Because MDI only measures the line-of-sight fluxes, the authors reconstructed the vertical flux under the assumption that the magnetic field in sunspots is vertical. The results showed a solar-cycle-like variation and only a minor long-term decrease in vertical magnetic flux of the active regions. Pevtsov *et al.* (2011) employed the manual measurements of magnetic field strengths taken in the Fe I $630.15-630.25$ nm wavelengths in the period of $1957-2010$ in seven observing stations that form the solar observatory network across eleven time zones in what is now Azerbaijan, Russian Federation, and in the Ukraine. To mitigate the differences in the atmospheric seeing and the level of the observers' experience, Pevtsov *et al.* (2011) considered only the strongest sunspot measurements for each day of observations. The sunspot field strengths were found to vary strongly with the solar cycle, and no

long-term decline was noted. Rezaei, Beck, and Schmidt (2012) used the magnetographic observations from the Tenerife Infrared Polarimeter at the German Vacuum telescope in the period of 1999 – 2011 to confirm the cycle variations of sunspot magnetic fields. Comparing maximum field strengths measured at the rising phases of cycles 23 and 24, the authors had noted a slight reduction in field strengths at the beginning of cycle 24. Still, the main variations in the magnetic field strength were found to be related to the solar cycle.

With the exception of Pevtsov *et al.* (2011), all previous studies based their conclusions on the data from the most recent full cycle 23. In our present article, we extend the analysis to earlier solar cycles. First, we use the data from the Mount Wilson Observatory (MWO) and apply the same technique as in Pevtsov *et al.* (2011). Next, we establish a correlation between the area of a sunspot and its field strength and use this correlation to construct a proxy for the magnetic field strength based on sunspot areas measured by the Royal Greenwich Observatory (RGO) and USAF/NOAA. The proxy for the magnetic field strength allows us to extend the analysis to cycles 11 – 24 and to scale the 1920 – 1958 MWO peak field measurements to those of Pevtsov *et al.* (2011) for the 1957 – 2010 interval. Our analysis is presented in Sections 2 and 3, and our results are summarized and discussed in Section 4.

2. Sunspot Field Strength Measurements from Mount Wilson Observatory (1920 – 1958)

To investigate the properties of the sunspot magnetic fields, we employed the observations from the MWO from May 1920 till December 1958. Specifically, we selected only data published in the *Publications of the Astronomical Society of the Pacific* (PASP). Although the sunspot field strength measurements continue to the present day (with a major interruption between September 2004 to January 2007 due to funding problems), their regular publication was discontinued at the end of 1958. This 1920 – 1958 part of the MWO data set should be considered as the most uniform; in later years there were several major changes to the instrumentation and the observing procedure (*e.g.*, multiple replacements of the spectrograph grating, selection of a new spectral line for measurements, and a different tilt-plate). In addition, the resolution of the field strength measurements changed from 100 gauss (G) through 1958 to 500 G for the later measurements. A summary of these changes can be found in Livingston *et al.* (2006).

We digitized the data summaries in PASP and verified the newly constructed tables against the published record. In a few instances, the MWO had issued corrections to the original tables (also published in PASP). All these corrections were taken into account in the process of data verification. Finally, we found a small number of inconsistencies in the original tables, which were also corrected.

The MWO sunspot summaries provide the MWO sunspot number, the sunspot's field strength (no polarity information), latitude, and an estimated date of the central meridian passage (later data also show the first and last days of an active region on the disk as well as its magnetic classification). In this study, we only use the sunspot field strength, latitude, and the date of the central meridian crossing. For some active regions the maximum field strength is estimated (see explanation in PASP **50**: "When it seems probable that the greatest field-strength observed in any group was not the maximum for that group, an estimated value is given in parentheses"). The field strength was measured manually using a glass tilt-plate. By tilting the plate, the observer co-aligned two Zeeman components of the spectral line, and the tilt angle translated into a linear (wavelength) displacement. Since the stronger field strengths require larger tilt angles, this procedure may introduce a non-linearity in the

Figure 1 Fractional distribution of sunspot field strengths for different cycles. The bin size is 400 G. Measurements with zero field strength are excluded.
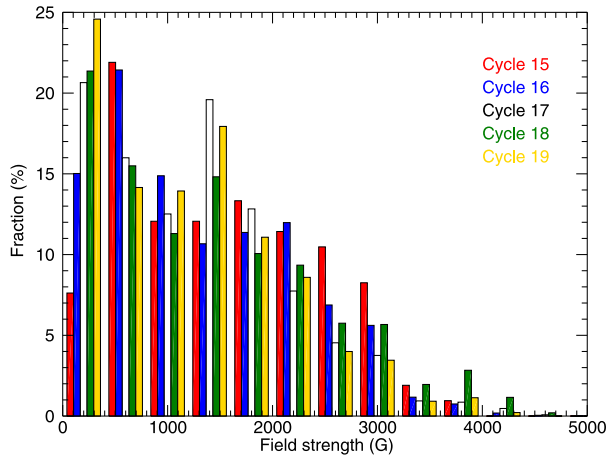
relation between the plate's tilt and the linear displacement. This slight non-linearity was corrected following the procedure specified in Livingston *et al.* (2006). Only measurements exceeding 2400 G were affected by this non-linearity.

A quick examination of the entire data set revealed several systematic effects. First, we found an increased number of values in parentheses (estimated values) in later years of the data set than in earlier years. Therefore, we excluded all estimated data from our investigation. Second, we noted a gradual increase in the fraction of measurements with weak field strengths from earlier to later years. For example, for observations taken during cycle 15 only 4 % of all measurements have field strengths of 100 G. For later cycles, the fraction increases significantly to 9 % (cycle 16), 15 % (cycle 17), 14 % (cycle 18), and 18 % (cycle 19). The tendency for a higher percentage of measurements with weaker fields is quite obvious in the annual number of measurements with zero fields (when observations were taken, but the fields were considered to be weaker than 100 G). Even after normalizing for the level of sunspot activity (using international sunspot numbers), the annual number of measurements with zero fields shows a steady (and significant) increase from 1920 to 1958.

Figure 1 shows the fractional distribution of sunspot field strengths in 400 G bins in the range of 0 – 5000 G. Data for different cycles are shown by different colors. The most significant differences are confined to the first two bins. The field strengths in the range of 100 – 400 G show a systematic increase in the fraction of weak field measurements from about 8 % in cycle 15 to about 25 % in cycle 19. These fractions are given in all measurements taken in a given cycle. The measurements in the next bin (500 – 800 G) show the opposite trend with a decrease in the fraction of measurements from about 22 % in cycles 15 and 16 to about 14 % in cycle 19. Stronger fields do not show any systematic behavior from one cycle to the next.

One can speculate that at least some of these systematic effects could be due to changes in the observing requirements (*e.g.*, increase in the "granularity" of measurements, when measurements are taken in separate umbrae of a single sunspot) and/or a "learning curve" effect (when with increasing experience the observer begins to expand the measurements to smaller sunspots). An increase in the scattered light might have similar effects on the visual measurements of the magnetic fields. These systematic effects may affect the average value of the field strengths. For example, including the larger number of measurements with the weaker field strengths will decrease the average value. To mitigate the negative effects of this possible change in statistical properties of the data sample, we employed an approach

similar to the one used by Pevtsov *et al.* (2011), where only the strongest measured field strength is selected for any given day of observations.

Next, we investigated the temporal behavior of the strongest sunspot field strengths. Since the published summaries of the MWO observations do not provide the date of measurements, we adopted the estimated date of the central meridian crossing as a proxy for the day of observations.

Applying the Pevtsov *et al.* (2011) approach to the MWO data is complicated by the fact that MWO measurements are, strictly speaking, not the daily observations. In many cases, there are daily drawings, but no corresponding magnetic field measurements. Also, when measurements do exist, they may exclude some (sometimes even the largest) sunspots that were present on the disk. For example, during a disk passage of a large sunspot (MWO AR 7688) on 17–28 November 1944, the magnetic field measurements were taken only on 17–19 November and 22 November. On 22 January 1957, the field strengths were measured only in smallest sunspots and pores; the largest sunspots were not measured. From 27 February–2 March 1942, the measurements alternate between the largest sunspots on the disk (one day) and the smallest sunspots (the following day). Since the published summaries of the MWO observations provide only the estimated date of the central meridian crossing (but not the date of observations), the selection of only one measurement of the strongest field strength for any given day of the observations as in Pevtsov *et al.* (2011) may not work well (*i.e.*, measurements of weaker field strengths are less likely to be excluded even if there are sunspots with stronger fields on the disk). Therefore, we modified the method by selecting only the measurement of the strongest field strength for any given week of observations.

Livingston *et al.* (2006) suggested that, since for field strengths weaker than 1000 G, the Zeeman splitting for Fe 617.33 nm becomes comparable to the Doppler width of the spectral line, the measurements of these fields are unreliable. However, a well-trained observer can consistently measure fields weaker than 1000 G. Livingston *et al.* (2006) relate one example where multiple measurements from the same observer agree within 10 G. We also examined several drawings in more detail and found a good persistence in day-to-day measurements of pores with field strengths below 1000 G. Nevertheless, to be cautious, we chose to exclude field strength measurements below 1000 G in our analysis. Still, including field strength measurements below 1000 G does not change the main results of this article.

Figure 2 (upper panel) shows the latitudinal distribution of active regions in our MWO data set and the monthly averages of the (weekly) strongest field strengths (Figure 2, lower panel). The most prominent trend in the data is a solar cycle variation with a minimum field strength in sunspots around the minima of solar cycles, and a maximum field strength near the maxima of solar cycles (see Table 1). To verify the presence (or absence) of the long-term trend, we fitted the data with linear and quadratic functions. The linear fit suggests a negligible decrease in sunspot field strengths over 40 years of about $0.8 \pm 1.7 \, \mathrm{G \, year^{-1}}$.

3. Sunspot Area as Proxy for Magnetic Field Strength

Several studies (*e.g.*, Houtgast and van Sluiters, 1948; Rezaei, Beck, and Schmidt, 2012) found a correlation between the sunspot areas (S, in millionths of solar hemisphere, MSH) and their recorded field strength (H_{MAX}). Ringnes and Jensen (1960) found the strongest correlation between the area logarithm and the field strength. Here, we use this relationship to investigate the changes in sunspot magnetic field strength over a long time-interval by employing the sunspot area as a proxy for the magnetic field strength. Figure 3 shows the relationship between the logarithm of the sunspot area and their field strength. Areas are

Figure 2 Time-latitude distribution of sunspots in the MWO data set included in our study (upper panel), and monthly average of daily peak sunspot field strengths (lower panel, dots connected by thin line). The thick gray line is the 18-point running average.
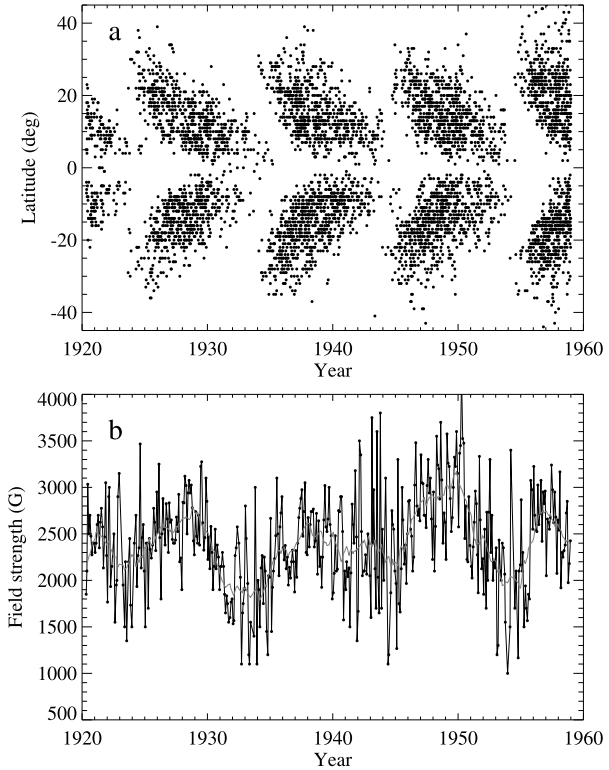

Table 1 Years of maxima and minima for solar cycle and sunspot field strengths.

Solar cycle		Sunspot field	
Minimum	Maximum	Minimum	Maximum
1923		1923.6	
	1928		1928.8
1933		1933.3	
	1937		1938.1
1944		1944.7	
	1947		1949.8
1954		1953.9	

taken from the independent data set of the RGO. For this plot, we established a correspondence between the active regions in the MWO and RGO data using the date of the central meridian passage and the latitude of active regions. Only regions with a small difference ($\leq 0.6°$) in latitude and less than 4.8 h (0.2 day) in the time of the central meridian crossing between the two data sets were included. We also excluded RGO areas smaller than 10 MSH. Solar features with area $S \leq 10$ MSH are small spots or pores; their field strengths are more likely to have large measurement errors.

Similar to previous studies, we find a reasonably good correlation between the (logarithm of) sunspot areas and the magnetic field strength (Pearson's correlation coefficient $\rho = 0.756$). Similar (strong) correlation coefficients were found for individual cycles (see

Figure 3 Magnetic field strength (from MWO observations) vs. the natural logarithm of the sunspot area (from RGO observations) for cycles 15–19. The dashed line is a first-degree polynomial fit to the data. Fitted coefficients are shown in Table 2, in the entry for "all cycles". For comparison, the solid line shows a fit by a quadratic function.
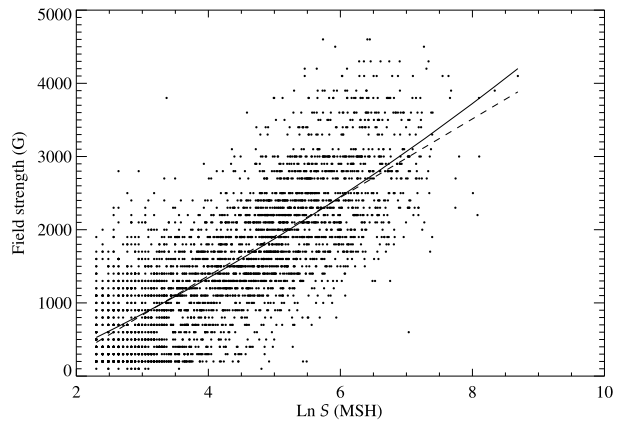
Table 2 Correlation (ρ) and fitted coefficients for the $H = A + B \times \ln(S)$ dependency between magnetic field strength (H) and the deprojected area (S) of an active region. Student's t-values and maximum sunspot number (SSN) for the $(n + 1)$-th cycle are also shown.

| Cycle No.	A	B	ρ	t-value	99 %-level	SSN_{n+1}
Cycle 15	-274.1 ± 177.6	507.3 ± 40.2	0.775	12.633	2.623	78.1
Cycle 16	-475.1 ± 63.4	514.9 ± 13.9	0.811	36.947	2.583	119.2
Cycle 17	-771.0 ± 59.9	523.2 ± 13.2	0.781	39.595	2.581	151.8
Cycle 18	-1106.9 ± 78.9	609.2 ± 16.9	0.739	35.966	2.580	201.3
Cycle 19	-800.4 ± 69.5	495.4 ± 14.9	0.784	33.252	2.583	110.6
All cycles	-774.2 ± 35.6	536.0 ± 7.7	0.756	69.170	2.577	
Cycles 16–18	-806.3 ± 41.0	551.7 ± 8.9	0.761	61.670	2.578	

Table 2). To verify the statistical significance of the correlations, we used the t-test; the t-values and the cutoff value for the 99 % confidence level are shown in Table 2. Since the t-values are well above the 99 % cutoff values, all correlations are statistically significant. The relationship between the magnetic field strength and the logarithm of the sunspot area is very similar for cycles 15–17. For cycle 18 the relation is "steeper" (sunspots with the same areas correspond to stronger magnetic fields than in cycles 15–17), and for cycle 19, the field-area relationship is somewhat weaker and more similar to the relationship found in cycle 15. Our data set only partially includes these two cycles. Cycles 16–18 are included in their entirety. Overall, we find that the MWO data for cycle 18 contain a slightly higher percentage of stronger field measurements than all other cycles (*e.g.*, see Figure 1). Limiting the data to three complete cycles 16–18 does not significantly affect the coefficients for the functional relation between sunspot area and magnetic field strength (see Table 2). Figure 3 indicates a non-linearity between the sunspot area and their magnetic field strength. On the other hand, a quadratic fit to the data (solid line, Figure 3) suggests that this non-linearity is small. For simplicity, in the following discussion we employ a linear relation between the logarithm of sunspot areas and their magnetic field strength.

Using the relation $H_{\text{MAX}} = -774.2 + 536.0 \times \ln(S)$ based on cycles 15–19 (Table 2), we created a proxy for the magnetic field strength based on the sunspot areas. The data used

Figure 4 Proxy of the magnetic field strength computed from the deprojected sunspot areas. Annual averages are shown as a thin solid line. The thick solid line is a second-degree polynomial approximation to the data, and the two thick dashed lines represent a one-standard deviation error band for the fit. The red line shows the magnetic field proxy derived using the quadratic dependency shown in Figure 3.
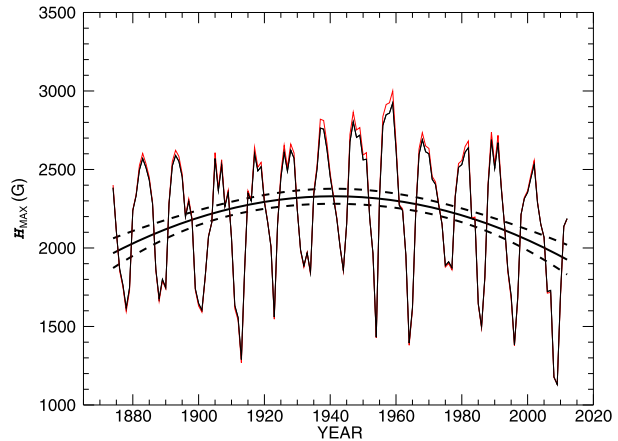

for this exercise represent a combination of the RGO measurements from 1874 – 1977 and the USAF/NOAA data from 1977 – early 2012. The USAF/NOAA data were corrected (by D. Hathaway) by a factor of 1.4 as described in http://solarscience.msfc.nasa.gov/greenwch.shtml. Figure 4 shows annual averages of the proxy of the magnetic field derived from the deprojected active region areas. For consistency with the scaling between the area logarithm and the magnetic field strength derived earlier, Figure 4 does not include areas smaller than 10 MSH. Including all areas does not noticeably change Figure 4. Similar to the direct measurements of the magnetic field, the proxy also traces the solar cycle variations with an approximate amplitude of 500 – 700 G. The data also show a long-term trend (see the parabolic fit to the data), with the mean value of the field proxy increasing from 1874 to around 1920 by about 300 G and then, following a broad relatively flat maximum to around 1960, decreasing by about same amount by early 2012. For comparison, Figure 4 also shows the magnetic field proxy computed using a quadratic fit to the area-field strength relation (red line).

Finally, using the magnetic field proxy as reference, we combined the MWO data and the Russian sunspot field strength observations from Pevtsov *et al.* (2011) into a single data set. Figure 5 shows two data sets on the same scale. The MWO data were re-scaled to the Russian data using the functional dependencies between the H_{MAX} (MWO) and the magnetic field proxy (sunspot areas) and a similar dependency for the Russian data. With the scaling, the mean level of approximately 2500 G in Figure 5 agrees with that observed at NSO/Kitt Peak from 1998 – 2005 by Penn and Livingston (2006).

The most prominent feature of the combined data shown in Figure 5 is the cycle variation of the sunspot field strength. Maxima and minima in the sunspot field strength agree relatively well with the maxima and minima in the sunspot number. The combined data set (Figure 5) shows no statistically significant long-term trend. A linear fit to the data (not shown in Figure 5) has a slope of $\approx -1.9 \pm 0.8$ G year^{-1}. This trend appears to be entirely due to the points corresponding to the deep minimum around 2010, and it disappears once these few points are excluded from the fitting (the fit shown in Figure 5 has a slope of $\approx -0.2 \pm 0.8$ G year^{-1}). In either case, the amplitude of the trend is significantly smaller than the -52 G year^{-1} reported by Penn and Livingston (2006). As the Penn and Livingston (2011) data cover the period of 1998 – 2011, the trend in the field strengths found in their data may be dominated by the declining phase of solar cycle 23.
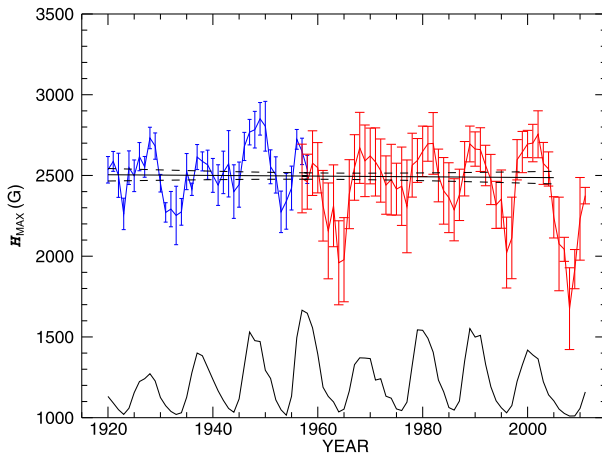
Figure 5 Annual averages of the magnetic field measurements from the MWO (blue color, up to 1958) and the Russian data set from Pevtsov *et al.* (2011, red). The MWO data are scaled to the Russian data set using the proxy of the magnetic field strength in Figure 4 as reference. Error bars show a $\pm 1\sigma$ standard deviation of the mean values. The thick black straight line is a linear approximation to the MWO and the Russian data. Dashed lines indicate the statistical uncertainties of this linear approximation. The black line in the lower part of the figure shows the annual international sunspot numbers.

4. Discussion

We employed observations of the sunspot field strengths from a subset of the MWO data covering solar cycles 15 – 19 (1920 – 1958). The data were analyzed using a modified approach of Pevtsov *et al.* (2011), where the strongest field measurement was selected for each week of observations. This approach allows one to mitigate the effects of some negative properties of the data set, for example, a systematic increase in the number of weak field measurements from the beginning of the data set to its end. Our findings confirm the presence of the 11-year cycle variation in the sunspot daily strongest field strengths similar to that found in the previous studies for solar cycles 19 – 23 (Pevtsov *et al.*, 2011; Watson, Fletcher, and Marshall, 2011; Rezaei, Beck, and Schmidt, 2012). On the other hand, no significant secular trend is found for the period covered by the MWO data (cycles 15 – 19). We do see a weak gradual decrease in average field strengths of $\approx 2.1 \pm 0.9$ G year^{-1}.

A correlation between the area of sunspots and the magnetic field strength allows us to use the sunspot area as a proxy for the magnetic field strength. With this approach, we showed that the cycle variations (in the magnetic field strengths as represented by their proxy) are present during cycles 11 – 24 (1874 – early 2012). The amplitude of these cycle variations is about 1000 G between the solar activity minima and maxima. The magnetic field strength proxy does show a secular trend: between 1874 and ≈ 1920, the mean value of the magnetic field proxy increased by about 300 G, and following a broad maximum in 1920 – 1960, it decreased by 300 G. The nature of this trend is unknown, but we note that the broad "maximum" in 1920 – 1960 includes the mid-twentieth century maximum of the current Gleissberg cycle, which began at about 1900. The long-term trend in the magnetic field strength proxy based on the sunspot area (Figure 4) could be subject to several uncertainties. For example, sunspot area estimates from the RGO observations prior to ≈ 1910 appear to be systematically lower than later data (Leif Svalgaard, private communication). Moreover, the functional dependency between the sunspot area and the peak magnetic field strength

may be non-linear and/or vary from one cycle to another (see the quadratic and linear fits in Figure 3). For example, a long-term variation in the $H_{MAX} - S$ dependency was found in Ringnes and Jensen (1960). The steepness of the linear regression in cycle n may correlate with the amplitude of cycle $(n + 1)$ (compare the B coefficients and the yearly international sunspot number, SSN_{n+1}, in Table 2), although our statistical sample is too small to be more conclusive with respect to this possible dependency. Nevertheless, the H_{MAX} proxies computed using linear and non-linear scaling functions agree reasonably well (compare black and red lines in Figure 4).

Using the proxy of H_{MAX} as a reference allows us to combine the MWO and Russian measurements of the magnetic field into a single data set. Although this data set does not show any statistically significant long-term trend comparable in amplitude with the trend reported in Penn and Livingston (2011), one can notice that the last three (see red curve in Figure 5) minima in the sunspot field strengths progressively decrease. The overall trend derived from this tendency is about 15 G year^{-1}, which could be associated with the Gleissberg cycle variations as derived from other studies (*e.g.*, Mordvinov and Kuklin, 1999). Alternatively, this could be an indication of some other long-term pattern in sunspot field strengths. One can note a similar tendency for progressively lower field strengths in the solar minima of cycles $12 - 14$ (Figure 4) and cycles $17 - 19$ (Figures 4 and 5). We leave the investigation of these patterns to future studies.

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