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REVERSALS OF GNEVYSHEV–OHL RULE

Andrey G. Tlatov
Kislovodsk Mountain Astronomical Station of Pulkovo Observatory, Gagarina Str. 100, Kislovodsk 357700, Russia; tlatov@mail.ru
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ABSTRACT

We perform an analysis of the number of sunspot groups in activity cycles from 1610 through the present. Here we use the G_n index, which is defined as the average daily number of sunspot groups in cycle n. There is a high positive correlation between the parameter G_n in the current cycle and an analogous parameter in the following cycle G_{n+1} both for pairs of even–odd cycles and odd–even activity cycles. In cycle Nos. 10–21 for pairs of even–odd cycles, the ratio of parameter G_n corresponds to the GO rule $G_{n+1}^{\rm odd}/G_n^{\rm even} > 1$. However, during the period ∼1745–1850, odd cycles were less than the preceding even cycles $G_{n+1}^{\rm odd}/G_n^{\rm even} < 1$. The ratio of the parameter $G_{n+1}^{\rm odd}/G_n^{\rm even}$ has a long-term variation within the range 0.5–1.5 with a period of about 21 activity cycles, and it proves the reversal of the GO rule.

Key words: Sun: activity – sunspots

1. INTRODUCTION

An empirical rule of Gnevyshev–Ohl (GO) is formulated for a pair of subsequent solar cycles. Gnevyshev & Ohl (1948) presented information about the analysis of the annual average values of a sunspot index R (Wolf number) for cycle Nos. −4 to 17. It was discovered that the sum of the index $\sum R$ in even cycles $2n$ has a good positive correlation with the succeeding odd cycle $2n+1$, while the correlation in pairs of an odd cycle $2n-1$ and an even cycle $2n$ is weak. It allowed the authors to formulate the hypothesis that the 22nd cycle begins with an even cycle with respect to small magnitude. Then, it is followed by an odd cycle, the magnitude of which is determined by the preceding cycle, and it indicates at close physical connection between them (Gnevyshev & Ohl 1948).

Presently, there are several definitions of the GO rule: (1) the amplitude of the even activity cycle is less than the amplitude of the following odd cycle; (2) the sum of the Wolf numbers in the even cycle is less than the sum of the following odd cycle; (3) the area under the curve of the Wolf numbers in the even cycle correlates with the area under the curve in odd cycles, at the same time the even and the following odd cycle form a pair (Kopecky 1950; Hathaway et al. 2002; Nagovitsyn et al. 2009; Ogurtsov & Lindholm 2011). The GO rule in its different definitions is justified for cycles 10–21, but there are some violations for pairs 4–5, 8–9, and 22–23 (Gnevyshev & Ohl 1948; Wilson 1988; Hathaway 2010).

Usually, to check the GO rule, one uses the sequence of Wolf numbers, reconstructed by Wolf since 1749 (Wolf 1861). However, this sequence has significant noise in earlier observations and does not take other kinds of observations into consideration (Hoyt et al. 1994; Hoyt & Schatten 1998; Svalgaard 2012). Based upon additional data, Hoyt et al. (1994) offered an index of sunspot group number, reconstructed in the period from 1610 through 1995. The index of a sunspot group number gives the best correlation ratio between the amplitudes of even and odd cycles in comparison with the Wolf number (Hathaway et al. 2002). In order to check the GO rule, Mursula et al. (2001) offered to use the index of sunspot group number as $I_{\rm GO}(k) = 1/132\sum_{j=J(k)}^{J(k+1)-1} Rg$, where Rg is the average monthly index of a sunspot group number, $J(k)$ is the month of the beginning of cycle k, and invariable 1/132 is the introduced scale of the obtained index toward standard indices of sunspots. The authors showed that during the period from 1725 through 1782, even cycles have larger amplitudes than the following odd cycles. To eliminate this discrepancy, they offered a hypothesis—one solar cycle was lost in the beginning of the Dalton minimum during the 1790s (Usoskin et al. 2001; Usoskin et al. 2009). The index $I_{\rm GO}(k)$ offered by the authors (Mursula et al. 2001) is analogous to the sum of the sunspots per cycle. However, in the case of a large gap in observation days, there is difficulty in calculating the sum of sunspot groups per cycle, as well as in defining the amplitude of a cycle.

This Letter offers to apply an index of a sunspot group number, based not upon summing the number of sunspot groups per cycle, but upon calculating the average number of sunspot groups per day during one activity cycle.

2. DATA AND ANALYSIS

To characterize the activity cycles, we can apply the daily average number of groups in a cycle: $G_n = \sum_{T_n}^{T_{n+1}} g/N_d$, where g is the amount of groups per day, N_d is the number of observation days in cycle n, and T_n is the moment of the beginning of cycle n. Preliminary data (Hoyt et al. 1994) contain information on different observatories and practicing astronomers about the number of sunspot groups per day (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/GROUP_SUNSPOT_NUMBERS/alldata.txt). Within this analysis, if there were series of observations, the one with the largest number of groups was chosen. The results were also checked by means of data with interpolate daily values. The data concerning the beginning of cycles T_n were also taken from the Web site NGDC. Table 1 shows the values of parameter G_n in the period from 1610 through the present. For cycles 23 and 24, the values are performed during the calculation of the number of groups in accordance with the sunspot data base USAF/NOAA (http://solarscience.msfc.nasa.gov/greenwch.shtml). Statistics of Wolf numbers have existed since 1749 (cycle No. 0). There is a positive correlation between index G_n and the amplitude of cycle in Wolf numbers with $W_n^{\max} = 45.0(12) + 19.0(3)G_n$; $r = 0.8$.

Figure 1 shows the time change in the ratio of the index of the group number in the following activity cycle G_{n+1}, toward the preceding cycle G_n, for cycle Nos. −12 to 23. Pairs $G_{n+1}^{\rm odd}/G_n^{\rm even}$ and $G_{n+1}^{\rm even}/G_n^{\rm odd}$ are presented with different symbols. After the

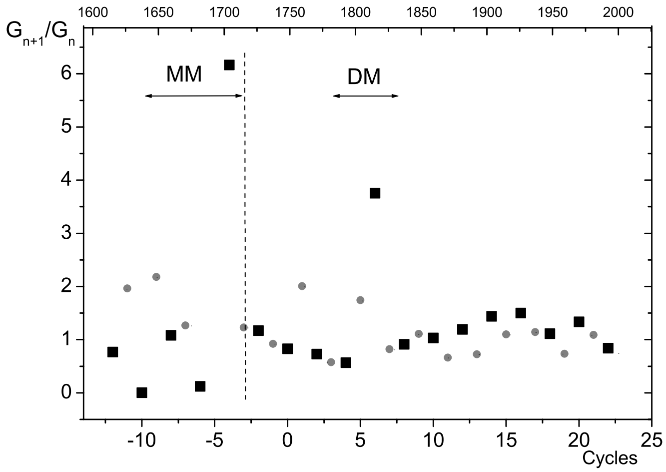

Figure 1. Ratio of the average daily number of sunspot groups in neighboring cycles $G_{n+1/}/G_n$. The squares indicate $G^{\rm odd}_{n+1}/G^{\rm even}_n$ pairs, the circles show $G^{\rm even}_{n+1}/G^{\rm odd}_n$. The positions of the Maunder Minimum (MM) and the Dalton Minimum (DM) are presented.

Table 1
Index of the Average Daily Number of Sunspot Groups

No. of Cycle	G_n	μ	N	G_{n+1}/G_n	$T_{\rm min}$
−12	1.614	0.071	1022	0.766	1610.8
−11	1.236	0.039	1469	1.966	1619
−10	2.43	0.069	1634	0.005	1634
−9	0.011	0.003	3209	2.182	1645
−8	0.024	0.002	3926	1.083	1655
−7	0.026	0.002	4270	1.269	1666
−6	0.033	0.003	3629	0.121	1679.5
−5	0.004	0.002	3378	...	1689.5
−4	0.188	0.008	3800	6.165	1698
−3	1.159	0.05	2772	1.23	1712
−2	1.426	0.072	819	1.169	1723.5
−1	1.667	0.137	150	0.926	1734
0	1.544	0.072	612	0.827	1745
1	1.277	0.041	1317	2.009	1755.2
2	2.565	0.054	1388	0.727	1766.5
3	1.866	0.076	614	0.578	1775.5
4	1.079	0.054	693	0.567	1784.7
5	0.612	0.025	2695	1.742	1798.3
6	1.066	0.024	3007	3.753	1810.6
7	4.001	0.086	3522	0.82	1823.3
8	3.279	0.058	2539	0.914	1833.9
9	2.997	0.041	4337	1.109	1843.5
10	3.323	0.043	4065	1.03	1856
11	3.424	0.048	4234	0.665	1867.2
12	2.277	0.032	3908	1.193	1878.9
13	2.717	0.037	4419	0.727	1889.6
14	1.975	0.029	4346	1.441	1901.7
15	2.846	0.044	3652	1.1	1913.6
16	3.131	0.042	3726	1.5	1923.6
17	4.696	0.06	3798	1.144	1933.8
18	5.374	0.069	3689	1.112	1944.2
19	5.974	0.08	3872	0.737	1954.3
20	4.402	0.048	4236	1.334	1964.9
21	5.871	0.076	3762	1.091	1976.5
22	6.405	0.09	3360	0.719	1986.8
23	5.39	0.057	4419	0.79	1996.9
24	4.28	0.076	1016		2009

Notes. Gn in activity cycles, confidence interval $\mu = \sigma N^{1/2}$, the number of observation days in the activity cycle n, the relation of average number of groups in the following cycle to the previous G_{n+1}/G_n, and the moment of activity cycles' minimum $T_{\rm min}$.

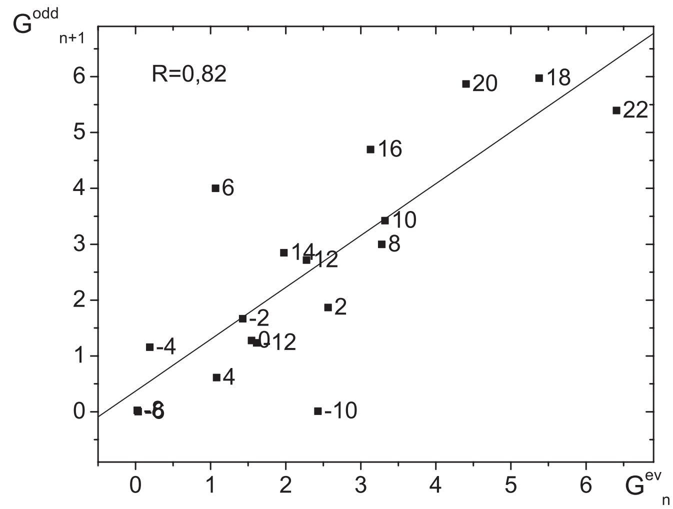

Figure 2. Average number of sunspot groups in the current cycle $G^{\rm even}_n$ compared with the number of sunspot groups in the following cycle $G^{\rm odd}_{n+1}$ for pairs of even–odd cycles.

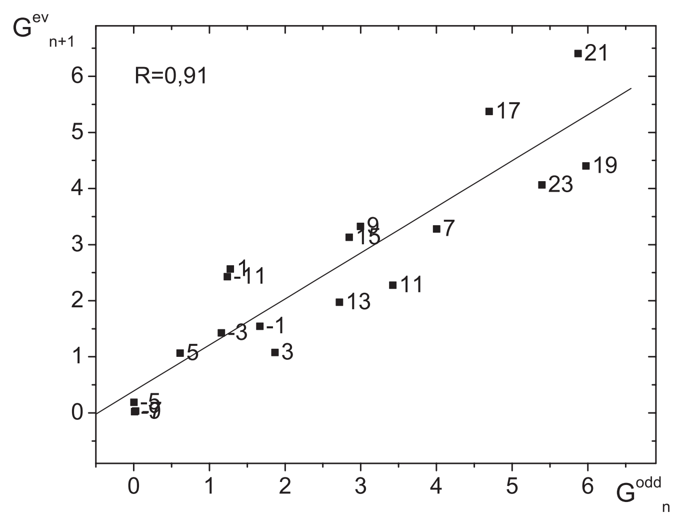

Figure 3. Same as Figure 2, but for pairs of odd $G^{\rm odd}_n$ and the following even activity cycles $G^{\rm even}_{n+1}$.

end of the Maunder Minimum (1645–1715), starting from cycle No. −2, for pairs of even and odd cycles, there is a ranking of values $G^{\rm odd}_{n+1}/G^{\rm even}_n$ in the form of a long-term modulation. The pair of cycle Nos. 6 and 7 represents an exception, taking place during the Dalton Minimum.

The average amount of sunspot groups in the following cycle G_{n+1} is linked with the number of groups in the preceding cycle G_n. Figure 2 depicts function of G_{n+1} against G_n for pairs of even and the following odd cycles. The relation between G_n indices in such pairs has a positive correlation $G^{\rm odd}_{n+1} = 0.37(0.47) + 0.93(0.16)G^{\rm even}_n$; $r = 0.82$, which corresponds to the standard GO rule. In addition, there is a high correlation for pairs of odd–even cycles (Figure 3). The relation between indices of such pairs comprised $G^{\rm even}_{n+1} = 0.39(0.4) + 0.82(0.08)G^{\rm odd}_n$; $r = 0.91$.

Table 1 shows that during period Nos. 10–21, the average number of groups in odd cycles was higher than in preceding even cycles, and the relation $G^{\rm odd}_{n+1}/G^{\rm even}_n$ corresponds to the standard formulations of the GO rule, but is violated in the pair of cycles 22–23. Figure 4 represents the relation $G^{\rm odd}_{n+1}/G^{\rm even}_n$ in the period after the Maunder Minimum. All pairs of cycles except cycles 6 and 7 are within the range of values 0.5–1.5.

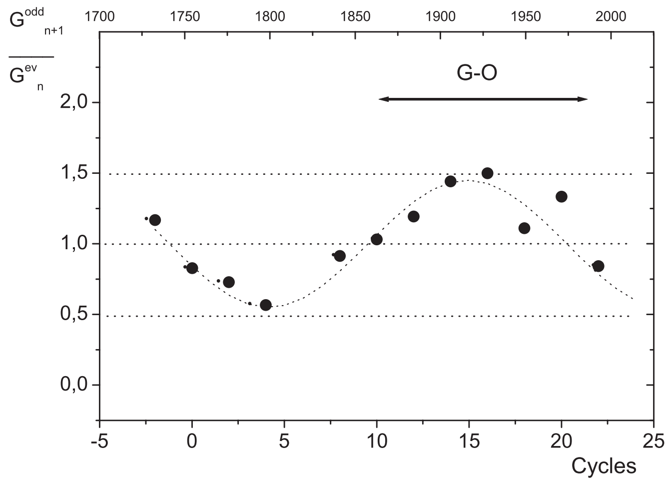

Figure 4. Ratio of the average daily amount of sunspot groups in the odd cycle to the analogous value in the preceding even cycle $G^{\rm odd}_{n+1}/G^{\rm even}_n$. An envelope line is drawn, and a line where this ratio is equal to 1.0, 0.5, and 1.5.

Starting from cycle No. -2, the relation $G^{\rm odd}_{n+1}/G^{\rm even}_n$ has a smooth envelope. As a comparison, the diagram shows the sinusoid with a period $t = 21$ cycles and an amplitude $a = 0.45$: $f(t) = 0.5 + 0.45 \cdot \sin(2\pi n/t)$. The standard deviation in a ratio $G^{\rm odd}_{n+1}/G^{\rm even}_n$ from the envelope curve amounted to $\sigma = 0.12$. The χ^2-test gives sinusoid a value of about 0.15 while for linear dependence it was about 0.69, which supports the proposed hypothesis.

3. DISCUSSION AND CONCLUSIONS

Research on the GO rule can provide important information about the nature of the solar periodicity, in particular concerning the possible fossil solar magnetic field, with which one usually connects this effect (Bravo & Stewart 1995; Charbonneau 2005). Some authors have proposed that the regularity when even cycles are less intensive than the following odd ones has a constant character (Usoskin et al. 2001; Nagovitsyn et al. 2009). However, the activity cycles 22–23 obviously show the violation of this rule (Hathaway 2010). Therefore, it is possible that this rule was reversed in previous centuries.

This paper uses G_n, the average daily number of sunspot groups in cycle n, to test the GO rule. We discovered that the ratio $G^{\rm odd}_{n+1}/G^{\rm even}_n$ in cycle Nos. 10–21 in pairs of even–odd cycles was more than 1, while cycles 22–23 showed less than 1, and it corresponds to the results obtained by means of the Wolf numbers (Hathaway 2010). A strong correlation of G_n parameters was found in pairs of even–odd cycles for all of the periods under consideration, cycle Nos. -12 to 23 ($r = 0.82$, Figure 2). These results correspond to the standard definitions of the GO rule and its exception for pair 22–23, and it indicates the validity of using the parameter G_n to test the GO rule. At the same time, a high positive correlation ($r = 0.91$) was found in pairs of odd–even activity cycles (Figure 3).

Applying index G_n allowed us to single out a long-term envelop curve for values $G^{\rm odd}_{n+1}/G^{\rm even}_n$ after the Maunder Minimum (Figure 1). The changes are close to long-term variations with a period of about $t \sim 21$ cycle (Figure 4) or about 230 yr. The only exception is cycles Nos. 6 and 7, because it takes place in Dalton minimum. Some authors (Vitinsky et al. 1986; Mursula et al. 2001) came to the conclusion that the apparition of the 22 yr periodicity disappeared in the time when the level of solar activity changed quickly, for instance, during the restoration of activity after the Maunder Minimum, or closer to the Dalton minimum. Long-term cyclicity with the period about $\sim$200–220 yr was found by means of reconstructing solar activity according to prior radioisotope data (Suess 1980; Mordvinov & Kramynin 2010; Abreu et al. 2012).

For the period of $\sim$1745–1850, the value of correlation in pairs of even–odd cycles $G^{\rm odd}_{n+1}/G^{\rm even}_n$ was less than 1. It proves that the GO rule can reverse within long intervals: to be more exact, even cycles can be stronger than the following odd cycles. The duration of epochs when $G^{\rm odd}_{n+1}/G^{\rm even}_n > 1$ and $G^{\rm odd}_{n+1}/G^{\rm even}_n < 1$ are approximately equal, and the reversal takes place during secular activity minimums (Figure 4). It is possible to expect that the following activity cycles will develop within the reversed GO rule.

Presumably, the violation in 22 yr cycles, when the ratio $G^{\rm odd}_{n+1}/G^{\rm even}_n$ becomes either more or less than 1 for a period of time, has a periodic character, during which the Sun changes its cycle mode. As a rule, one can observe minima of century variations of solar activity in the process.

To explain this, we can assume that in long-term periods there is a permanent solar magnetic field which can also reverse, and it reverses the sequence of 22 yr cycles. Such a permanent field appears because of the so-called magnetic memory under the bottom generation zone (Tlatov 1996). This field appears during the averaging of the magnetic fields of several subsequent cycles, having different directions of the poloidal field, thus ensuring the relation G_{n+1}/G_n will be higher (lower) than 1 during long-term periods (Figure 1). The positive correlation between the preceding and following cycle $G^{\rm odd}_{n+1}/G^{\rm even}_n$ and $G^{\rm even}_{n+1}/G^{\rm odd}_n$ (Figures 2 and 3), as well as changes with long-term period, which are depicted in Figure 1, count in favor of this hypothesis, as well as long-term changes, visible in Figures 1 and 4.

The average number G_n of sunspot groups in cycle n, unlike the total number R_n of sunspots, shows high correlation in the ratio G_{n+1}/G_n in both pairs of even–odd and odd–even cycles. In this way, applying index G_n differs from the standard GO rule. It is possibly connected with changes of the number of spots in sunspot groups in activity cycles. At the same time, only pairs of even and odd cycles show long-term changes (Figure 4). Thus, there are differences in the 22 yr magnetic cycle for pairs of even–odd and odd–even cycles, and there are differences in index G_n.

The violation of the GO rule in activity cycles 22–23 can be a sign of change in the character of the periodicity period and long-term reversal of the GO rule in the following activity cycles. The GO rule, established and corrected for cycles 10–21, is a part of long-period inequality of solar activity.

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